

## Gyökös egyenletek, egyenletrendszerek

1.  $\sqrt[3]{x+1} = \sqrt{x-3}$
2.  $\sqrt{x+8} - \sqrt{5x+20} + 2 = 0$
3.  $\sqrt{3x^2+5x+8} - \sqrt{3x^2+5x+1} = 1.$
4.  $\sqrt{x+3} - 4\sqrt{x-1} + \sqrt{x+8} - 6\sqrt{x-1} = 1.$
5.  $\sqrt[3]{(a+x)^2} + 4\sqrt[3]{(a-x)^2} = 5\sqrt[3]{a^2-x^2}.$
6.  $\sqrt[3]{x} + \sqrt[6]{x} - 2 = 0.$
7.  $5\sqrt[4]{x} + 2 = 3\sqrt{x}.$
8.  $x^5 - 33x^2\sqrt{x} + 32 = 0.$
9.  $x^3 - 3x\sqrt{x} + 2 = 0.$
10.  $2\sqrt[3]{x} + 5 = 63\sqrt[3]{\frac{1}{x}}.$
11.  $2x\sqrt[3]{x} - 3x\sqrt[3]{\frac{1}{x}} = 20.$
12.  $x^2 + 11 + \sqrt{x^2 + 11} = 42.$
13.  $x^2 - \sqrt{x^2 - 9} = 21.$
14.  $\frac{x^3 + (a^2 - x^2)\sqrt{a^2 - x^2}}{x + \sqrt{a^2 - x^2}} = a^2.$
15.  $\sqrt{(x-2)^2} + \sqrt{(x+1)^2} = \sqrt{(x+2)^2}, x > 0.$
16.  $\sqrt{x^2 - 4x + 4} - \sqrt{x^2 - 6x + 9} = \sqrt{x^2 - 2x + 1}.$
17.  $\frac{\sqrt{4-x} + \sqrt{x-3}}{\sqrt{4-x} - \sqrt{x-3}} = \frac{2}{3}\sqrt{\frac{4-x}{x-3}}.$
18.  $\sqrt{1+x} + \sqrt{1-x} = 1.$
19.  $\sqrt{23 + \sqrt{2x + \sqrt{5x^2 - 21x - 68}}} = 5.$
20.  $x = a + \sqrt{a^2 + x\sqrt{x^2 - a^2 - b^2}}.$
21.  $\sqrt{1 + x\sqrt{x^2 - 24}} = x - 1.$
22.  $\frac{1}{x} + \frac{1}{a} = \sqrt{\frac{1}{a^2} + \sqrt{\frac{1}{a^2x^2} + \frac{1}{x^4}}}.$
23.  $x^2 + 5x + 4 = 5\sqrt{x^2 + 5x + 28}.$
24.  $(x+5)(x-2) + 3\sqrt{x(x+3)} = 0.$
25.  $x^2 + 4x - 8\sqrt{8x} + 20 = 0.$
26.  $x^2 - 3x - 6\sqrt{3x} + 18 = 0.$
27.  $x^2 - 3x - 2\sqrt{2x} + 6 = 0.$
28.  $x^{10} - x^5 - 2\sqrt{x^5} + 2 = 0.$
29.  $x^2 - 3x - 5\sqrt{9x^2 + x - 2} = 2,75 - \frac{28}{9}x.$
30.  $\sqrt{x+5} - 4\sqrt{x+1} + \sqrt{x+2} - 2\sqrt{x+1} = 1.$
31.  $\sqrt{4x+2} + \sqrt{4x-2} = 3.$
32.  $\sqrt{4-x} + \sqrt{5+x} = 3.$
33.  $\sqrt{25-x} + \sqrt{9+x} = 2.$
34.  $\sqrt{1+x+x^2} + \sqrt{1-x+x^2} = 4.$
35.  $\sqrt{x^2+x+1} = \sqrt{x^2-x+1} + 1.$
36.  $x\sqrt{x} + \sqrt{x} - 2 = 4(\sqrt{x} - 1).$
37.  $x^2 + 2(x+1)\sqrt{x} + 3x = 8.$
38.  $x^3 + 4x\sqrt[3]{(x-1)^2} + 3x^2 - 8x + 4 = 0.$
39.  $x^6 - x^3 - 2x^2 - 1 = 2(x - x^3 + 1)\sqrt{x}.$
40.  $(x+2)^2 + 2(x+2)\sqrt{x} - 3\sqrt{x} - 2x = 46.$
41.  $2(x + \sqrt{x^2 - 1}) = (x - 1 + \sqrt{x+1})^2.$
42.  $x(x - 2\sqrt{x-1}) = 2\sqrt{x-1} - 3x.$
43.  $(x-1)[1 - x(1 + 2\sqrt{x})] = x^3 - (x-b)^2.$
44.  $2x + 1 + x\sqrt{x^2 + 2} + (x+1)\sqrt{x^2 + 2x + 3} = 0.$

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45.  $a^3 + 2(x-a) = 3a\sqrt[3]{(x-a)^2}$ .
46.  $\sqrt{a-\sqrt{a+x}} = x$ .
47.  $x = a + \sqrt{a+\sqrt{x}}$ .
48.  $\sqrt{x} + \sqrt{x-\sqrt{1-x}} = 1$ .
49.  $\sqrt{5+x+4\sqrt{x+1}} = 2 + \sqrt{x+1}$ .
50.  $\sqrt{x+2+2\sqrt{x+1}} + \sqrt{x+2-2\sqrt{x+1}} = 2$ .
51.  $\sqrt{x-\sqrt{x-2}} + \sqrt{x+\sqrt{x-2}} = 2$ .
52.  $\sqrt{4x-3} + \sqrt{5x+1} = \sqrt{15x+4}$ .
53.  $\sqrt{x^2+9} + \sqrt{x^2-9} = \sqrt{7} + 5$ .
54.  $\sqrt{(x-1)(x-2)} + \sqrt{(x-3)(x-4)} = \sqrt{2}$ .
55.  $\sqrt{2x^2+3x+5} + \sqrt{2x^2-3x+5} = 3x$ .
56.  $\sqrt{x+5} + \sqrt{x+3} = \sqrt{2x+7}$ .
57.  $\sqrt{x(1+\sqrt{x})} - \sqrt{x(1+x)} = \sqrt{1+x} - \sqrt{1+\sqrt{x}}$
58.  $(1-\sqrt{\sqrt{x}+1})\sqrt{\sqrt{x}+1} = \sqrt{x}$ .
59.  $(1+x)\sqrt{1+x} - (1-x)\sqrt{1-x} = x$ .
60.  $2(x-1) = (\sqrt{x}-1)(\sqrt{2-x}+1)$ .
61.  $\frac{1}{4}x = (\sqrt{1+x}-1)(\sqrt{1-x}+1)$ .
62.  $x\sqrt{x} + \sqrt{x+2} + \sqrt{x^2+2x} = 3$ .
63.  $\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$ .
64.  $\sqrt[3]{54+\sqrt{x}} + \sqrt[3]{54-\sqrt{x}} = \sqrt[3]{18}$ .
65.  $\sqrt[3]{(8-x)^2} + \sqrt[3]{(27+x)^2} = \sqrt[3]{(8-x)(27+x)} + 7$
66.  $bx\sqrt{a+x} + ab\sqrt{a+x} = a\sqrt{x^3}$ .
67.  $\sqrt{\sqrt{x} + \sqrt[3]{x\sqrt{a}}} + \sqrt{\sqrt{a} + \sqrt[3]{a\sqrt{x}}} = \sqrt[4]{b}$ .
68.  $\sqrt[3]{(a+x)^2} - \sqrt[3]{a^2-x^2} + \sqrt[3]{(a-x)^2} = b$ .
69.  $\sqrt[3]{a+x} - \sqrt[3]{a+\sqrt{x}} + \sqrt[3]{a-x} - \sqrt[3]{a-\sqrt{x}} = 0$ .
70.  $\sqrt[3]{1+\sqrt{x}} = 2 - \sqrt[3]{1-\sqrt{x}}$ .
71.  $\sqrt{x+\sqrt[3]{x^2-x^3}} + \sqrt{1-x+\sqrt[3]{x(1-x)^2}} = 1$ .
72.  $\sqrt[3]{(a+x)^2} + 4\sqrt[3]{(a-x)^2} = 5\sqrt[3]{a^2-x^2}$ .
73.  $\sqrt[3]{a+x} + \sqrt[3]{a-x} = \sqrt[6]{a^2-x^2}$ .
74.  $\sqrt{x^2+\sqrt[3]{x^4a^2}} + \sqrt{a^2+\sqrt[3]{a^4x^2}} = b$ .
75.  $\sqrt[3]{(1+x)^2} - (\sqrt[3]{1+x}-1)\sqrt[3]{1+\sqrt[3]{1+x}} = 1$ .
76.  $\sqrt[4]{a+x} + \sqrt[4]{a-x} = 2\sqrt[8]{a^2-x^2}$ .
77.  $\sqrt[n]{(x+1)^2} + \sqrt[n]{(x-1)^2} = 4\sqrt[n]{x^2-1}$ .
78.  $\sqrt[n]{(x+a)^3} + 2\sqrt[n]{x^3} = 3\sqrt[n]{x^2(x+a)}$ .
79.  $(1+\sqrt[3]{x})\sqrt[3]{x^2} + (1+\sqrt[3]{a})\sqrt[3]{a^2} = 2\sqrt[3]{ax}(1+\sqrt[6]{ax})$ .
80.  $\sqrt[5]{(3x-5)^3} - \sqrt[5]{(5-3x)^3} = -\frac{52}{10}$ .
81.  $(\sqrt[7]{x-1} + \sqrt[7]{x+1})^2 + 5\left[\sqrt[7]{(x-1)^2} - \sqrt[7]{(x+1)^2}\right] + 6(\sqrt[7]{x-1} - \sqrt[7]{x+1})^2 = 0$ .
82.  $(\sqrt[4]{x+a} + \sqrt[4]{x-a})^3 (\sqrt[4]{x+a} - \sqrt[4]{x-a}) = 2b$ .
83.  $\frac{\sqrt[7]{12+x}}{x} + \frac{\sqrt[7]{12+x}}{12} = 21\frac{1}{3}\sqrt[7]{x}$ .
84.  $\frac{\sqrt[n]{a+x}}{a} + \frac{\sqrt[n]{a+x}}{x} = \frac{\sqrt[n]{x}}{b}$ .
85.  $\frac{\sqrt{3-x} + \sqrt{x-2}}{\sqrt{3-x} - \sqrt{x-2}} = \frac{1}{5-2x}$ .

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$$86. \frac{\sqrt[4]{5-x} + \sqrt[4]{x-2}}{\sqrt[4]{5-x} - \sqrt[4]{x-2}} = \frac{2}{3} \sqrt[4]{\frac{5-x}{x-2}}.$$

$$87. \frac{x-1}{\sqrt{x+1}} = 4 + \frac{\sqrt{x}-1}{2}.$$

$$88. 1 + \sqrt{1 - \frac{a}{x}} = \sqrt{1 + \frac{x}{a}}.$$

$$89. \frac{\sqrt{2} - \sqrt{x}}{2-x} = \sqrt{\frac{1}{2-x}}.$$

$$90. \frac{1}{\sqrt{3x+10}} + \frac{6}{(x+2)(3x+10)} = \frac{1}{\sqrt{x+2}}.$$

$$91. \sqrt{x-a} - \sqrt{\frac{a^2}{a+x}} = \sqrt{2a+x}.$$

$$92. \frac{\sqrt{x+2a} - \sqrt{x-2a}}{\sqrt{x-2a} + \sqrt{x+2a}} = \frac{x}{a}.$$

$$93. \frac{1-ax}{1+ax} \sqrt{\frac{1+bx}{1-bx}} + 1 = 0.$$

$$94. \frac{x(\sqrt{x-1})^3 \sqrt{x-1}}{x - (\sqrt{x+1})} - \frac{x^2 - 2x\sqrt{x} + x - 1}{x - (\sqrt{x-1})} = 2.$$

$$95. \frac{a(x+a) + a\sqrt{x^2-a^2}}{x - \sqrt{x^2-a^2} + a} = \sqrt{x^2-a^2} + x\sqrt{x}.$$

$$96. \frac{\sqrt{1+\sqrt{x}} + \sqrt{x}}{\sqrt{1-\sqrt{x}} + \sqrt{x}} + \frac{\sqrt{1-\sqrt{x}} + \sqrt{x}}{\sqrt{1+\sqrt{x}} + \sqrt{x}} = 2.$$

$$97. \frac{\sqrt{x^2+x+6} + \sqrt{x^2-x-4}}{\sqrt{x^2+x+6} - \sqrt{x^2-x-4}} = 5.$$

$$98. \sqrt[n]{\frac{a-x}{b+x}} + \sqrt[n]{\frac{b+x}{a-x}} = 2.$$

$$99. \frac{\sqrt[m]{1+x^2} + \sqrt[m]{1-x^2}}{\sqrt[m]{1+x^2} - \sqrt[m]{1-x^2}} = \frac{p}{q}.$$

$$100. \frac{x}{\sqrt{1-x}+1} + \frac{x}{\sqrt{1+x}-1} = 1.$$

$$101. \sqrt{x+\sqrt{x}} - \sqrt{x-\sqrt{x}} = \frac{3}{2} \sqrt{\frac{x}{x+\sqrt{x}}}.$$

$$102. \frac{\sqrt{a+x}}{\sqrt{a} + \sqrt{a+x}} = \frac{\sqrt{a-x}}{\sqrt{a} - \sqrt{a-x}}.$$

$$103. \frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}} + \frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^2+1} + \sqrt{x^2-1}} = 4\sqrt{x^2-1}.$$

$$104. \sqrt{x - \frac{1}{x}} - \sqrt{1 - \frac{1}{x}} = \frac{x-1}{x}.$$

$$105. \frac{a-x}{\sqrt{a} + \sqrt{a-x}} + \frac{a+x}{\sqrt{a} + \sqrt{a+x}} = \sqrt{a}.$$

$$106. \frac{1+x - \sqrt{2x+x^2}}{1+x + \sqrt{2x+x^2}} = a^3 \frac{\sqrt{2+x} + \sqrt{x}}{\sqrt{2+x} - \sqrt{x}}.$$

$$107. \sqrt{12 - \frac{12}{x^2}} + \sqrt{x^2 - \frac{12}{x^2}} = x^2.$$

$$108. \begin{cases} \frac{7}{\sqrt{x-7}} - \frac{4}{\sqrt{y+6}} = \frac{5}{3}; \\ \frac{5}{\sqrt{x-7}} + \frac{3}{\sqrt{y+6}} = 2\frac{1}{6}. \end{cases}$$

$$109. \begin{cases} \sqrt{x} + \sqrt{y} = 3; \\ xy = 4. \end{cases}$$

$$110. \begin{cases} \sqrt[3]{x} + \sqrt[3]{y} = 3; \\ xy = 8. \end{cases}$$

$$111. \begin{cases} \sqrt[3]{x} - \sqrt[3]{y} = 2; \\ xy = 27. \end{cases}$$

$$112. \begin{cases} x = 6\sqrt{x+y}, \\ y = 2\sqrt{x+y}. \end{cases}$$

$$113. \begin{cases} (x^2 + xy + y^2)\sqrt{x^2 + y^2} = 185, \\ (x^2 - xy + y^2)\sqrt{x^2 + y^2} = 65. \end{cases}$$

$$114. \begin{cases} \sqrt[4]{x^3} + \sqrt[5]{y^3} = 35, \\ \sqrt[4]{x} + \sqrt[5]{y} = 5. \end{cases}$$

$$115. \begin{cases} x + y = 10, \\ \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2}. \end{cases}$$

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$$116. \begin{cases} x + y - \sqrt{x} + \sqrt{y} - 2\sqrt{xy} = 2, \\ \sqrt{x} + \sqrt{y} = 8. \end{cases}$$

$$117. \begin{cases} \sqrt{\frac{x+y}{5x}} + \sqrt{\frac{5x}{x+y}} = \frac{34}{15}, \\ x + y = 12. \end{cases}$$

$$118. \begin{cases} x + y - \sqrt{\frac{x+y}{x-y}} = \frac{12}{x-y}, \\ xy = 15. \end{cases}$$

$$119. \begin{cases} \sqrt{\frac{3y-2x}{y}} + \sqrt{\frac{4y}{3y-2x}} = 2\sqrt{2}, \\ 3(x^2+1) = (y+1)(y-x+1). \end{cases}$$

$$120. \begin{cases} x + y + \sqrt{xy} = 14, \\ x^2 + y^2 + xy = 84. \end{cases}$$

$$121. \begin{cases} \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 1 + \frac{7}{\sqrt{xy}}, \\ \sqrt{x^3y} + \sqrt{xy^3} = 78. \end{cases}$$

$$122. \begin{cases} x^2 + y\sqrt{xy} = 420, \\ y^2 + x\sqrt{xy} = 280. \end{cases}$$

$$123. \begin{cases} x\sqrt{x} + y\sqrt{y} = 341, \\ x\sqrt{y} + y\sqrt{x} = 330. \end{cases}$$

$$124. \begin{cases} \sqrt[3]{\frac{x+y}{x-y}} - \sqrt[3]{\frac{x-y}{x+y}} = \frac{3}{2}, \\ x^2 - y^2 = 32. \end{cases}$$

$$125. \begin{cases} \sqrt[3]{6x+5} - \sqrt[3]{4x-3y} = 1, \\ 6x+3y = 4. \end{cases}$$

$$126. \begin{cases} x^3 + xyz = \sqrt{xyz}, \\ y^3 + xyz = \sqrt{xyz}, \\ z^3 + xyz = \sqrt{xyz}. \end{cases}$$

$$127. \begin{cases} \sqrt{x} + \sqrt{y} + \sqrt{z} = 4, \\ x + y + z = 6, \\ x^2 + y^2 + z^2 = 18. \end{cases}$$

$$128. \begin{cases} \sqrt{x} + \sqrt{y} = z, \\ 2x + 2y + a = 0, \\ z^4 + az^2 + b = 0. \end{cases}$$

$$129. \begin{cases} \sqrt{x+y} + \sqrt{y+z} = 3, \\ \sqrt{y+z} + \sqrt{z+x} = 5, \\ \sqrt{z+x} + \sqrt{x+y} = 4. \end{cases}$$

$$130. \begin{cases} \sqrt{x+\frac{1}{y}} + \sqrt{y+\frac{1}{x}} = 2\sqrt{2}, \\ (x^2+1)y + (y^2+1)x = 4xy. \end{cases}$$

$$131. \begin{cases} x + \sqrt{y} - 56 = 0, \\ \sqrt{x} + y - 56 = 0. \end{cases}$$

$$132. \begin{cases} \sqrt[3]{x+2y} + \sqrt[3]{x-y+2} = 3, \\ 2x+y = 7. \end{cases}$$

$$133. \begin{cases} \sqrt{\frac{20y}{x}} = \sqrt{x+y} + \sqrt{x-y}, \\ \sqrt{\frac{16x}{5y}} = \sqrt{x+y} - \sqrt{x-y}. \end{cases}$$

$$134. \begin{cases} \sqrt[3]{\frac{y+1}{x}} - 2\sqrt[3]{\frac{x}{y+1}} = 1, \\ \sqrt{x+y+1} + \sqrt{x-y+10} = 5. \end{cases}$$

$$135. \begin{cases} \sqrt{x^2+y^2} + \sqrt{x^2-y^2} = 6, \\ xy^2 = 6\sqrt{10}. \end{cases}$$

$$136. \begin{cases} \sqrt{x} + \sqrt{y} = 3, \\ \sqrt{x+5} + \sqrt{y+3} = 5. \end{cases}$$